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1995 J. Phys. A: Math. Gen. 28 5689

(http://iopscience.iop.org/0305-4470/28/19/023)

J. Phys. A: Math. Gen. **28** (1995) 5689–5691. Printed in the UK

COMMENT

Addendum to 'A unified formulation of the spectra of temperature fluctuations in isotropic turbulence'

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Received 22 March 1995

Abstract. In this note, we consider further the unified formulation of the spectra of temperature fluctuations in isotropic turbulence given by us previously. We indicate restrictions imposed on the model parameters by general physical considerations.

Shivamoggi and Andrews (1991) developed a generalized Heisenberg–von Weizsacker type model which describes the temperature spectra in all the subregimes of wavenumbers in a unified way through a single formula. The purpose of this note is to make further considerations of this unified formulation and to indicate restrictions imposed on the model parameters by general physical considerations.

For the inertial-convective range, the Shivamoggi–Andrews formulation gave the well known result for the scalar spectrum function

$$\Gamma(k) \sim k^{-5/3} \tag{1}$$

whereas for the viscous-convective range, the Shivamoggi–Andrews formulation led to

$$\Gamma(k) \sim k^{-\frac{5}{3}+\left(\frac{m-2}{n-1}-\frac{5}{3}\right)/\left(\frac{3}{2n}-1\right)}. \tag{2}$$

Noting that the energy spectrum in the viscous regime satisfies

$$E(k) \sim k^{-(m-2)/(n-1)} \qquad \frac{m-2}{n-1} > \frac{5}{3} \tag{3}$$

and that in the viscous-convective regime we require

$$\Gamma(k) \sim k^{-q} \qquad q < \tfrac{5}{3} \tag{4}$$

reflecting the bump in the temperature spectrum in this regime, as experimentally observed by Williams and Paulson (1977) and Champagne *et al* (1977), we have from (2)

$$0 < n < 1 \qquad \text{and} \qquad m < \tfrac{1}{3}(5n+1)$$

or

$$1 < n < \tfrac{3}{2} \qquad \text{and} \qquad m > \tfrac{1}{3}(5n+1). \tag{5}$$

For the inertial-diffusive range, the Shivamoggi–Andrews formulation gave the following result:

$$\Gamma(k) \sim k^{-\frac{5}{3}n+m-2}. \tag{6}$$

Noting that, in this regime, we require

$$\Gamma(k) \sim k^{-q} \qquad q > \tfrac{5}{3} \tag{7}$$

we have from (6)

$$m < \tfrac{1}{3}(5n+1). \tag{8}$$

Equation (5), in conjunction with (8), now gives the admissible values for the model parameters m and n:

$$0 < n < 1 \qquad \text{and} \qquad m < \tfrac{1}{3}. \tag{9}$$

For the viscous-diffusive regime, the Shivamoggi–Andrews formulation gave the following result:

$$\Gamma(k) \sim k^{-(m-2)/(n-1)}. \tag{10}$$

Observe that the range of admissible values for m and n given in (9) automatically ensures from (10) that in this regime

$$\Gamma(k) \sim k^{-q} \qquad q > \tfrac{5}{3} \tag{11}$$

as required!

For the case $m = -\frac{3}{2}$ and $n = \frac{1}{2}$, which are compatible with (9) and correspond to a Heisenberg–von Weizsacker type model for the convective transfer of scalar variance, (2), (6) and (10) reduce to

$$\text{viscous-convective:} \quad \Gamma(k) \sim k \tag{12}$$

$$\text{inertial-diffusive:} \quad \Gamma(k) \sim k^{-\frac{13}{3}} \tag{13}$$

$$\text{viscous-diffusive:} \quad \Gamma(k) \sim k^{-7}. \tag{14}$$

It is to be noted that (13) and (14) were also given by Kerstein (1991) who used a stochastic simulation with a linear-eddy approach to model a scalar field advected by a turbulent velocity field.

It is of interest to note that the result of Batchelor (1959) for the viscous-convective regime, namely

$$\Gamma(k) \sim k^{-1} \tag{15}$$

and that of Batchelor *et al* (1959) for the inertial-diffusive regime, namely

$$\Gamma(k) \sim k^{-\frac{17}{3}} \tag{16}$$

can be obtained from (2) and (6) in a unified way for the following particular choice of the model parameters m and n,

$$m = -3.37 \qquad \text{and} \qquad n = 0.18 \tag{17}$$

which, of course, belongs to the set of admissible values of m and n given by (9)!

The authors are thankful to Dr Alan Kerstein for bringing his 1991 paper to their attention.

References

Batchelor G K 1959 *J. Fluid Mech.* **5** 113
Batchelor G K, Howells I D and Townsend A A 1959 *J. Fluid Mech.* **5** 134
Champagne F H, Friehe C A, LaRue J C and Wyngaard J C 1977 *J. Atmos. Sci.* **34** 515
Kerstein A R 1991 *J. Fluid Mech.* **231** 361
Shivamoggi B K and Andrews L C 1991 *J. Phys. A: Math. Gen.* **24** 4721
Williams R M and Paulson C A 1977 *J. Fluid Mech.* **83** 547